Chapter 9: Center of Mass and Momentum
Tuesday February 24th

- Review: Linear momentum and Newton's 2nd law
- Review: Momentum conservation
- Perfectly elastic collisions
- Inelastic collisions
- Example problems, *i*clicker and demos

Mid-term Exam next Tuesday:

- Full class period – 1hr 15 mins
- Cumulative – will cover everything up to LONCAPA #12
- I will discuss more on Thursday

Reading: up to page 150 in Ch. 9 (skip Ch. 8 for now)

Review: Linear momentum and Newton's 2nd Law

- Definition of linear momentum, $\vec{p}$:

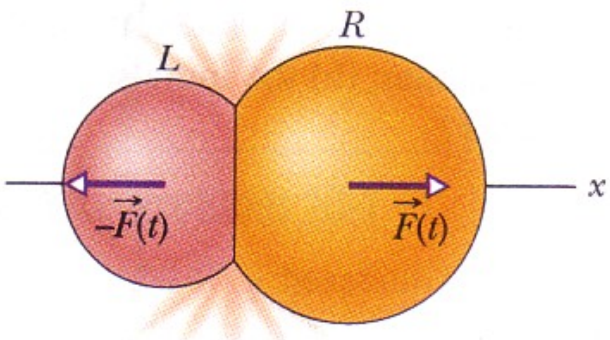
$$\vec{p} = m\vec{v}$$

- If one takes the derivative (constant mass):

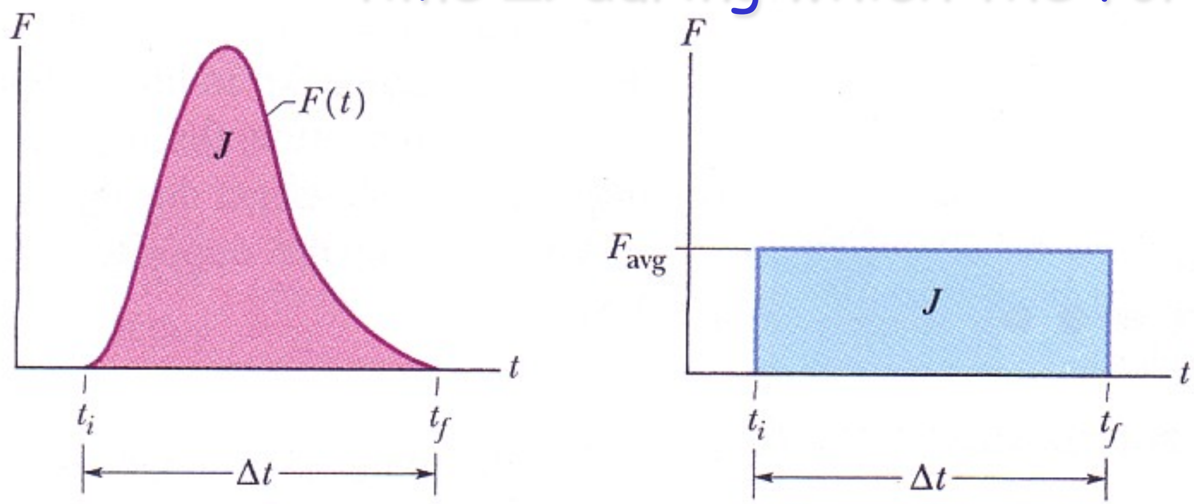
$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$$

The time rate of change of momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

Impulse and linear momentum



- The figure left shows a third law force pair between to bodies.
- The change in linear momentum depends not only on the force, but also on the time Δt during which the force acts.



- Definition:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt = \vec{F}_{\text{avg}}\Delta t = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

(impulse – linear momentum theorem)

CHANGE IN LINEAR MOMENTUM = IMPULSE

Conservation of linear momentum

- For a system of *n* particles, if no net force acts on the system:

$$\vec{P}_{\text{total}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots.. + \vec{p}_n = \text{constant} \qquad \text{(isolated system)}$$

If no net external force acts on a system of particles, the total linear momentum of the system cannot change

$$\begin{pmatrix} \text{total linear momentum} \\ \text{at some initial time } t_i \end{pmatrix} = \begin{pmatrix} \text{total linear momentum} \\ \text{at some later time } t_f \end{pmatrix}$$

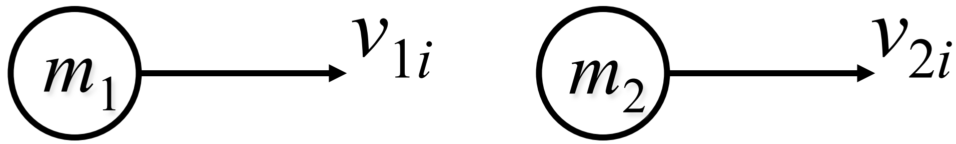
- These are vector equations, *i.e.*

$$P_x = \text{constant}; \quad P_y = \text{constant}; \quad P_z = \text{constant}$$

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Perfect elastic collision - general 1D case

Before:



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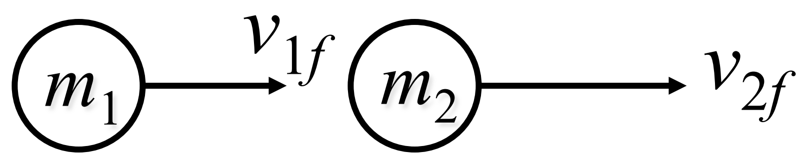


Conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

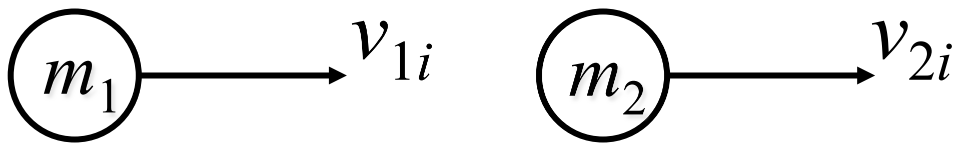
Conservation of kinetic energy (if perfectly elastic):

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

SIMULTANEOUS EQUATIONS

Perfect elastic collision - general 1D case

Before:



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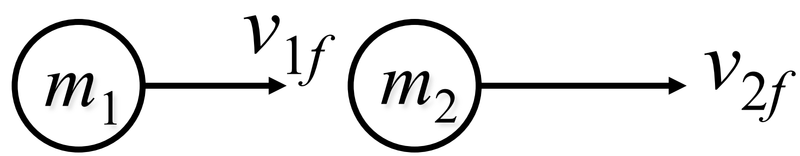


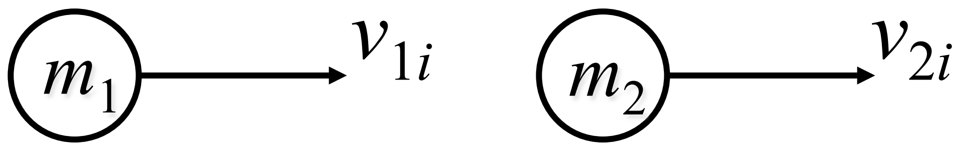
General result:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Perfect elastic collision - general 1D case

Before:



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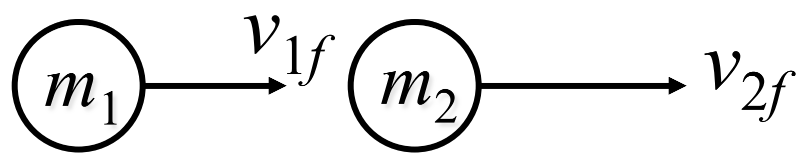


Special case: $$v_{2i} = 0$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

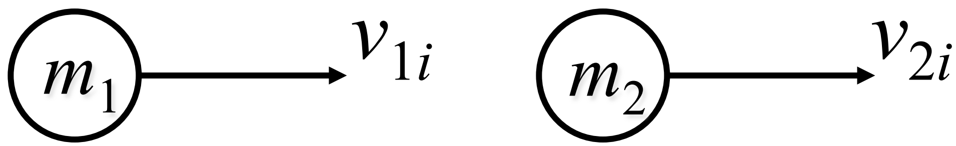
By definition:

$$K_f = K_i$$

(Starting assumption)

Inelastic collision - general 1D case

Before:



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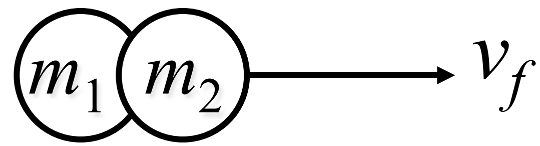


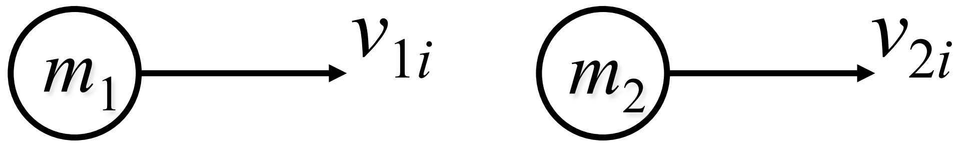
$$\Rightarrow \qquad v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Special case: $v_{2i} = 0$

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

Inelastic collision - general 1D case

Before:



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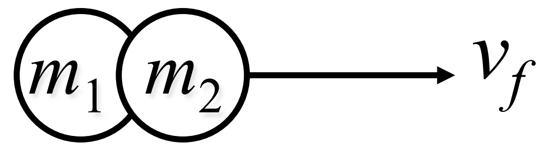


Special case: $v_{2i} = 0$

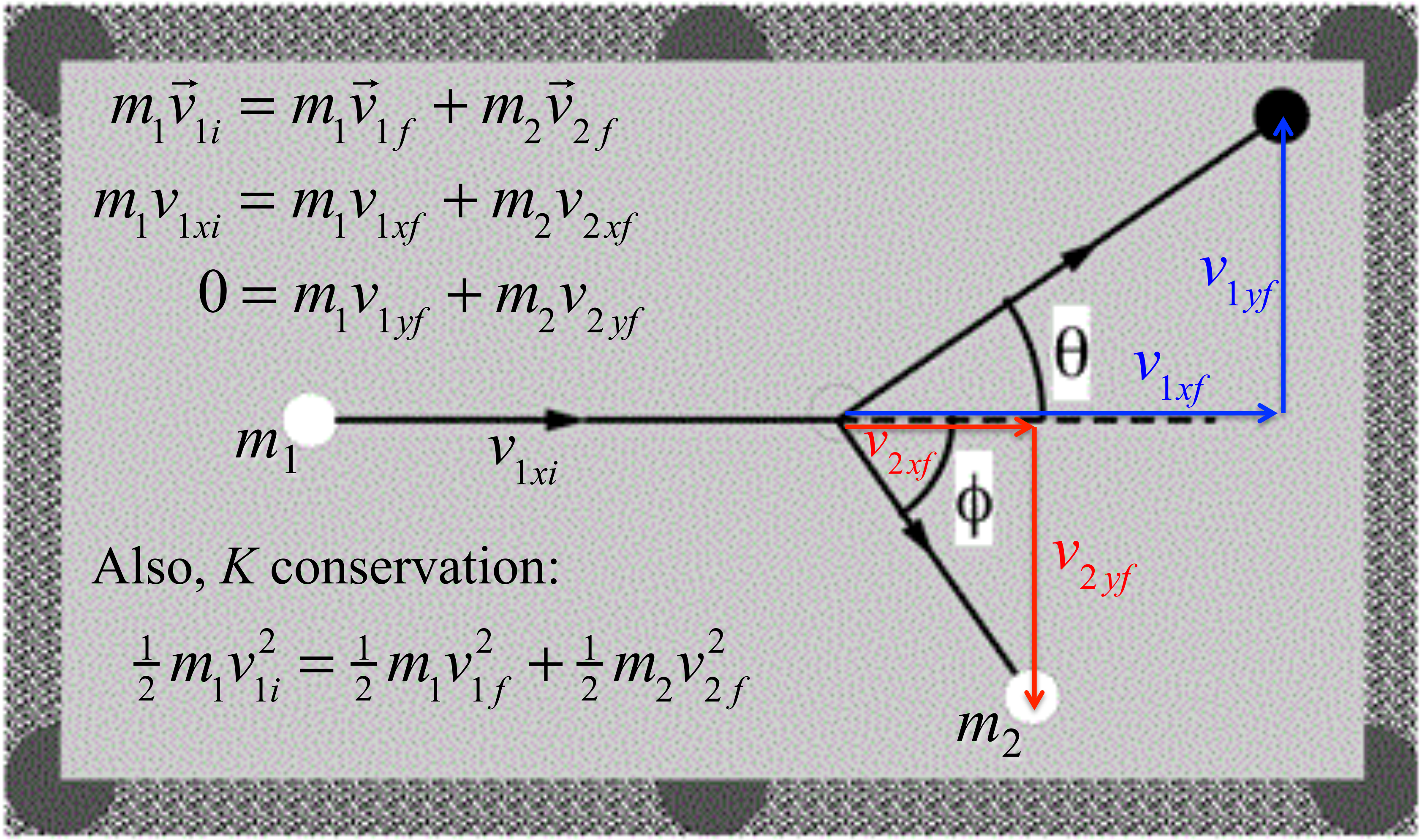
$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

Energy conservation:

$$K_f = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\right)^2 v_{1i}^2 = \left(\frac{m_1}{m_1 + m_2}\right)\tfrac{1}{2} m_1 v_{1i}^2 < K_i$$

Elastic Collision - 2D case

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.



$$m_1\vec{v}_{1i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$m_1 v_{1xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$0 = m_1 v_{1yf} + m_2 v_{2yf}$$

Also, *K* conservation:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$